Possible evidences from $H(z)$ parameter data for physics beyond ΛCDM

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We analyse $H(z)$ parameter data with some conditions by using Lagrange mean value theorem in Calculus. We find that: (1) there exists at least one decelerated phase at 1 σ confidence level in the redshift range (0.38, 0.59); (2) the equation of motion of dark energy may be less than $-1$ at 1 σ confidence level at some redshifts in the redshift range (1.3, 1.53); (3) there exists at least one accelerated phase at 1 σ confidence level in the redshift range (1.037, 1.944). These results may provide possible evidences for physics beyond ΛCDM.

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I. INTRODUCTION

A great number of independent cosmological observations, such as supernova Ia (SNIa) at high redshift [1, 2], large-scale structure [3], and the cosmic microwave background anisotropy [4, 5], have confirmed that the Universe is experiencing an accelerated expansion. In order to explain this phenomenon, an unknown energy component (dubbed as dark energy) usually have to be introduced in the framework of general relativity. The simplest and most theoretically sound scenario of dark energy is the vacuum energy (ΛCDM) with a constant equation of state (EoS) $w_x = p_x/\rho_x = -1$ where $w_x$ denotes the EoS of dark energy. This model is consistent with most of the current astronomical observations, but suffers from the cosmological constant problem [6] and age problem [7] as well. Recently, Hubble tension may also provide evidences for physics beyond ΛCDM [8].

The general approach to studying dark energy is to assume either a theoretical model or an EoS, and then use observational data to limit relevant parameters, see for example, for spatially-flat ΛCDM the Hubble constant and the matter density parameter are constrained as: $H_0 = (67.4 \pm 0.5) \, \text{km s}^{-1} \text{Mpc}^{-1}$, $\Omega_m = 0.315 \pm 0.007$, respectively; while for EoS ($w_x = w_0 + \frac{w_a}{1+z}$) parameterized model, the related parameters are limited as: $H_0 = (68.31 \pm 0.82) \, \text{km s}^{-1} \text{Mpc}^{-1}$, $w_0 = -0.957 \pm 0.080$, and $w_a = -0.29^{+0.32}_{-0.26}$ [5]. Statistical methods, such as the maximum likelihood [7, 9–12], are generally used to analyze the observational data to fit the parameters. These statistical method yields the best statistical results, but it is easy to eliminate some interesting (possibly important) data. Here we propose a model-independent method by using the Lagrange mean value theorem to analyze $H(z)$ parameter data. We find that the EoS of dark energy may be less than $-1$ at some redshifts and the accelerated phase may occur earlier than we previously thought.

The paper is organized as follows. In the next Section, we will present $H(z)$ parameter data and derive the equations needed to analyze these data. In Sec. III, We will provide the data and results obtained from the analysis. Finally, we will briefly summarize and discuss our results in section IV.

II. THEORETICAL METHOD AND $H(z)$ PARAMETER DATA

In this Section, we will present 63 $H(z)$ parameter data obtained recently, then introduce the Lagrange mean value theorem and combine it with Friedmann equations to derive equations needed to analyze $H(z)$ parameter data.

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A. Theoretical method

Assuming a Friedmann-Robertson-Walker-Lemaitre (FRWL) spacetime

\[ ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - K r^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \]

where \( a(t) \) is the scale factor, \( K \) denotes the curvature of the space with \( K = +1, 0, \) and \(-1\) corresponding to a closed, flat and open universe, respectively. We use the unit \( c = 1 \) here. According to the Planck 2018 results, the spacetime is spatially flat: \( \Omega_{K0} = 0.001 \pm 0.002 \) [5]. So we consider a spatially flat FRWL spacetime here, the Friedmann equations take the form

\[ H^2 = \frac{8\pi G}{3} \rho, \]

\[ \frac{\dot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p), \]

or equivalently

\[ \dot{H} = -4\pi G (\rho + p), \]

where the \( H \equiv \dot{a}/a \) is the Hubble parameter with the dot denoting the derivative with respect to the cosmic time \( t \). The total energy density \( \rho \) and pressure \( p \) contain contributions coming from the radiation, nonrelativistic matter, and other components. Because \( dz = -(1 + z)H dt \), we have

\[ \dot{H} = -(1 + z)H \frac{dH}{dz}. \]

Combining Eqs. (4) and (5), yields

\[ \frac{dH}{dz} = \frac{4\pi G}{(1+z)H} (\rho + p) = \frac{4\pi G \rho + (1 + w_i)}{(1+z)H}, \]

where \( w_i \) is the total EoS. From this equation, we can judge whether the total EoS is greater than, equal to, or less than \(-1\): see for example, if \( dH/dz < 0 \), we have \( w_x \leq w_k \leq -1 \) because of the positive of \( H \) and \( \rho \). In an era dominated by dark energy, we can also determine with Eq. (6) wether the EoS of dark energy is equal to \(-1\): if \( dH/dz = 0 \), then one have \( w_k \simeq w_i = -1 \). If \( dH/dz \leq 0 \), we know the Universe is experiencing an accelerated expansion. But if \( dH/dz > 0 \), we can’t judge whether the Universe speeds up. At this point, we need another important physical quantity, the deceleration parameter, which is defined as

\[ q = -1 + (1 + z) \frac{1}{H} \frac{dH}{dz}, \]

Now a question naturally rise: if we have some \( H(z) \) parameter data, how can we use them to directly determine \( dH/dz \) or \( q \)? Think of Lagrange mean value theorem in Calculus, which states: for a continuous and differentiable function \( f(x) \), there exists \( x_1 < x_{12} < x_2 \) satisfying

\[ \frac{df}{dx} \bigg|_{x=x_{12}} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}. \]

Applying this theorem to Hubble parameter which we assume is a continuous and differentiable function of \( z \), and taking function \( H(z) \) as \( f(x) \) in (8), we have

\[ H'(z_{ij}) \equiv \frac{dH}{dz} \bigg|_{z=z_{ij}} = \frac{H(z_i) - H(z_j)}{z_i - z_j}, \]

where \( z_j < z_{ij} < z_i \). If \( z_i - z_j \ll 1 \), \( H'(z_{ij}) \) will be large in general, which will make relevant results less credible. Since \( z_{ij} \) and \( H(z_{ij}) \) in Eq. (7) are unknown, we take approximatively: \( z_{ij} = (z_i + z_j)/2 \) and \( H(z_{ij}) \simeq [H(z_i) + H(z_j)]/2 \), which can be called as mid-value approximate method. Then we have

\[ q(z_{ij}) \simeq -1 + \frac{(2 + z_i + z_j) H'(z_{ij})}{H(z_i) + H(z_j)}. \]

Combining Eqs. (4) and (5), yields

\[ \frac{dH}{dz} = \frac{4\pi G}{(1+z)H} (\rho + p) = \frac{4\pi G \rho + (1 + w_i)}{(1+z)H}, \]

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where \( z_j < z_{ij} < z_i \). If \( z_i - z_j \ll 1 \), \( H'(z_{ij}) \) will be large in general, which will make relevant results less credible. Since \( z_{ij} \) and \( H(z_{ij}) \) in Eq. (7) are unknown, we take approximatively: \( z_{ij} = (z_i + z_j)/2 \) and \( H(z_{ij}) \simeq [H(z_i) + H(z_j)]/2 \), which can be called as mid-value approximate method. Then we have

\[ q(z_{ij}) \simeq -1 + \frac{(2 + z_i + z_j) H'(z_{ij})}{H(z_i) + H(z_j)}. \]
If \( z_i - z_j \) is large, in general, Eq. (10) will be not valid. Taking the uncertainty on the values of \( H(z) \) into account, the uncertainties associated to the \( H'(z) \) and \( q(z) \) are given by, respectively

\[
\sigma_{H'} = \sqrt{\frac{\sigma_{H_i}^2 + \sigma_{H_j}^2}{z_i - z_j}}, \quad (11)
\]

and

\[
\sigma_q = \frac{2(2 + z_i + z_j)H_i \sqrt{\sigma_{H_i}^2 + \sigma_{H_j}^2}}{(H_i + H_j)^2} \frac{1}{z_i - z_j}. \quad (12)
\]

With \( \sigma_{H'} \) and \( \sigma_q \), we can determine whether the results are credible at 1 \( \sigma \) confidence level.

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<th>( \sigma_H )</th>
<th>Reference Method</th>
<th>index ( z )</th>
<th>( H(z) )</th>
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**TABLE I:** Hubble parameter compilation from cosmic chronometers (DA) or from the radial BAO surveys (clustering).
TABLE II: \(H'(z)\) and \(q(z)\) data obtained from \(H(z)\) parameter data.

B. \(H(z)\) parameter data

The data set we use consists of 1 \(H(z)\) measurement from SNIa observation, 34 \(H(z)\) measurements obtained by calculating the differential ages of galaxies, which is called cosmic chronometer, and 28 \(H(z)\) measurements inferred from the baryon acoustic oscillation (BAO) peak in the galaxy power spectrum, as listed in Table I. In these cases, the datasets are given with their 1\(\sigma\) confidence interval.

III. APPLICATIONS

In this Section, we apply Eqs. (9), (10), (11), and (12) to investigate the evolution of the Universe with the observational Hubble parameter data. In order to avoid significant errors, we have considered the following limitations in the process of analyzing the \(H(z)\) data: 0.1 \(\leq z_i - z_j \leq 0.5\), \(\sigma_H \leq 5\) if \(H \leq 100\), and \(\sigma_H \leq 20\) if \(H \geq 100\). The data for \(H'(z)\), \(q(z)\) at 1\(\sigma\) confidence level are listed in Table II, from which we can conclude:

(a) At redshifts \(z_{41}, z_{71}, z_{111}, z_{141}, z_{171}, z_{181}, z_{201}, z_{251}, z_{311}, z_{341}, z_{371}, z_{38}, z_{279}, z_{259}, z_{279}, z_{319}, z_{339}, z_{2014}, z_{2714}, z_{259}, z_{249}, z_{4324}, z_{5150}, z_{5550}, z_{5650}, z_{5552}, z_{5652} z_{5554}, z_{5654},\) and \(z_{5854}\), the Universe experiences an accelerated expansion at 1\(\sigma\) confidence level.

(b) At redshifts \(z_{3418}, z_{3420}, z_{3718}, z_{3431}, z_{3727},\) and \(z_{3731}\), the Universe experiences a decelerated expansion at 1\(\sigma\) confidence level.

(c) At redshifts \(z_{3139}, z_{2014}, z_{5150}, z_{5550}, z_{5650}, z_{5552},\) and \(z_{5854}\), since \(H'(z) < 0\), implying \(w_X \leq w_t < -1\), but not at 1\(\sigma\) confidence level. However, we can infer that \(w_X \leq w_t < -1\) at redshifts \(z_{5652}, z_{5554},\) and \(z_{5654}\) at 1\(\sigma\) confidence level.

According to the Planck 2018 results [5], the matter density parameter for the spatially-flat \(\Lambda\)CDM was constrained as: \(\Omega_m = 0.315 \pm 0.007\), implying that the phase transition from deceleration to acceleration of the Universe occurs at the redshift \(z \approx 0.632\). Result (b), however, shows that there exists at least one decelerated phase in the redshift range \((0.38, 0.59)\) at 1\(\sigma\) confidence level. In addition, results (a) and (c) also indicate that the EoS of dark energy may be less than \(-1\) at some redshifts and the accelerated phase may occur earlier than we previously thought.
IV. CONCLUSIONS AND DISCUSSIONS

Using the Lagrange mean value theorem in Calculus, we have analysed $H(z)$ parameter data with some conditions and find that: (1) there exists at least one decelerated phase at 1 $\sigma$ confidence level in the redshift range (0.38, 0.59); (2) the EoS of dark energy may be less than $-1$ at 1 $\sigma$ confidence level at some redshifts in the redshift range (1.3, 1.53); (3) there exist at least one accelerated phase at 1 $\sigma$ confidence level in the redshift range (1.037, 1.944). The $q(z)$ data we obtained can be used to investigate cosmological models. The results may provide clues to address Hubble tension and possible evidences for physics beyond $\Lambda$CDM.

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